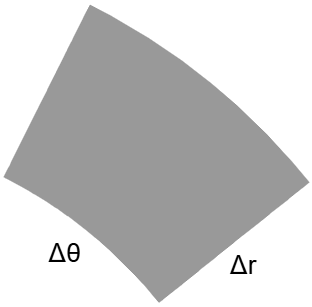
Double Integrals in Polar Coordinates

1. Let D be a disc of some fixed radius. Divide the disc into curved regions R bounded by arcs subtending angles $\Delta\theta$ and consisting of points with magnitude between $j\Delta r$ and $(j+1)\Delta r$. Find exactly or approximate the area of R to include both changes, $\Delta\theta, \Delta r$.



2. Use the area of small regions in D found in question 1 to rewrite the double integral of $f(x,y)$ over a disc D

$$\iint_D f(x,y)\,dA$$

in terms of the parametrization $x = r\cos\theta, y = r\sin\theta$. *Hint: Riemann sums.*

3. Find the volume of the region bounded below by the paraboloid $z = x^2 + y^2$ and above by the cone $z = 2 - \sqrt{x^2 + y^2}$.

4. Find the volume of the region bounded by $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$ over the annulus $R = \{(r, \theta) : 1 \leq r \leq 2, 0 \leq \theta \leq \pi\}$.

5. Find the area in of the crescent R (in $\mathbb{R}^2$) that is the region outside the circle $r = 2$ and inside the circle $r = 4\sin\theta$.

6. The average value of a function f over a region R is defined as

$$\overline{f} = \frac{1}{\text{Area}(R)} \iint_R f(x, y)\, dA.$$

Find the average value of the y-coordinates of the points in the semicircular disc of radius a give by $R = \{(r, \theta) : 0 \leq r \leq a, 0 \leq \theta \leq \pi\}$.